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STEADY THERMOMAGNETOHYDRODYNAMIC FLOWS OF A CONDUCTING GAS IN STRONG MAGNETIC FIELDS WITH ALLOWANCE FOR THE HALL EFFECT

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This article examines slow steady flows of a conducting gas in an arbitrary region when the magnetic forces and the pressure gradient in the main part of the flow are an order of magnitude greater than the inertial forces and viscous stresses and Ohm's law is written in its most general form with allowance for heat flow and the Hall effect. We will assume that we can ignore the distortion of the magnetic field of the moving medium. The flows to be studied have a high value of the MHD interaction parameter, a high Hartmann number, and a low magnetic Reynolds number.

The main equations which describe the motions of a conducting medium under the given conditions have the following form:
the continuity equation

$$\operatorname{div} \rho \mathrm{v} = 0; \tag{1}$$

the equation of magnetic statics

$$\operatorname{grad} p = \frac{1}{c}(\mathrm{j} \times \mathrm{H}); \tag{2}$$

the generalized Ohm's law in the presence of heat flows

$$\mathrm{j} = \sigma(\operatorname{grad} \varphi + \frac{\mathrm{v}}{c} \times \mathrm{H} - \alpha \operatorname{grad} T) - \gamma(\mathrm{j} \times \mathrm{H}) + \varkappa(\mathrm{j} \times \mathrm{H}) \times \mathrm{H}; \tag{3}$$

the law of charge conservation

$$\operatorname{div} \mathrm{j} = 0; \tag{4}$$

the energy equation

$$\rho c_v \frac{dT}{dt} = k\Delta T - p \operatorname{div} \mathrm{v} + \frac{j^2}{\sigma}; \tag{5}$$

the equation of state

$$p = p(\rho, T). \tag{6}$$

Here, ρ is the density of the medium; σ is electrical conductivity; v is velocity; p is pressure; c is the speed of light; j is current density; H is the strength of the magnetic field; φ is electric potential; k is thermal conductivity; c_v is isobaric heat capacity; T is temperature; α is the thermo-emf coefficient; the coefficients γ and $\varkappa$ are determined by the physical properties and state parameters of the gas. The conditions under which we can ignore the induced magnetic field, viscous forces, and inertial forces in the equations of motion are given by the inequalities

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$$H \gg \frac{1}{c} jL, \; M = \sqrt{\frac{\sigma}{\mu}} \frac{HL}{c} \gg 1, \; \rho v^2 \ll p,$$

where M is the Hartmann number; μ is viscosity; H, j, L, ρ, p, and v are the magnetic field, current density, a linear dimension, the density of the medium, pressure, and velocity.

The magnetic field will be assumed to be a potential field and will be determined by the assigned external sources H = grad a (where a is a known function satisfying the equation $\Delta a = 0$). We use Eq. (2) to find the component of current density perpendicular to the magnetic field [1]

$$\mathbf{j}_\perp = \frac{c\mathbf{H}}{H^2} \times \text{grad } p. \tag{7}$$

Here, pressure is a function which is constant on each magnetic line of force.

Following [1], we determine the component of current density in the direction of the magnetic field from Eq. (4) in the form

$$j_\parallel = H \int \frac{c}{H^2} (\mathbf{H} \times \text{grad } H^{-2}) \text{grad } p da + AH \tag{8}$$

(A is an arbitrary function which remains constant along the magnetic lines of force).

We find from Eq. (3) that

$$\sigma(\varphi - \alpha T) = c \int\int H^{-2} (\mathbf{H} \times \text{grad } H^{-2}) \text{grad } p da + Aa + B \tag{9}$$

(B is another arbitrary function which is constant on each magnetic line of force).

We find the component of velocity perpendicular to the magnetic field from Eq. (3) with allowance for Eq. (7):

$$\mathbf{v}_\perp = \left(-\frac{c^2}{\sigma H^2} - \frac{c^2}{\sigma}\varkappa\right) \text{grad } p - \frac{cH}{H^2} \times \text{grad}\left(\varphi - \alpha T - \gamma \frac{c}{\sigma} p\right). \tag{10}$$

We use Eq. (1) to calculate the velocity component in the direction of the magnetic field:

$$\begin{aligned} v_\parallel = \frac{H}{\rho} \int \frac{c^2}{H^2} \text{rot} \left(\frac{\rho \mathbf{H}}{H^2}\right) \times \text{grad}\left(\varphi - \alpha T - \gamma \frac{c}{\sigma} p\right) da + \\ \frac{H}{\rho} \int \left(\frac{c^2}{\sigma H^4} + \frac{c^2}{\sigma H^2}\varkappa\right) \rho \Delta p da + \frac{H}{\rho} \int \left(\frac{\rho c^2}{\sigma H^2} + \frac{\rho c^2}{\sigma}\varkappa\right) \text{grad } p \text{ grad } H^{-2} da + \\ \frac{H}{\rho} \int \left(\frac{c^2}{\sigma H^4} + \frac{c^2}{\sigma H^2}\varkappa\right) \text{grad } \rho \text{ grad } p \, da + \frac{H}{\rho} C \end{aligned} \tag{11}$$

(C is an arbitrary function that is constant on each magnetic line of force).

We now derive the equations of motion in the boundary layer with allowance for the Hall effect and heat flow. Along with the electromagnetic forces, thermal effects, the anisotropy of conductivity, and the pressure gradient, we will consider viscosity in the boundary layer. As before, inertial forces will be ignored. We will classify a boundary layer of the given type as a Hartmann layer.

The equations describing the flow in the boundary layer take the form

$$\begin{aligned} \mu \frac{\partial^2 \mathbf{v}}{\partial n^2} = \text{grad}\,(p + \varkappa H^2 p) + \frac{\sigma}{c} \mathbf{H} \times \text{grad}\left(\varphi - \alpha T - \frac{c}{\sigma} p\gamma\right) + \\ \frac{\sigma \mathbf{H}^2}{c^2} \mathbf{v} - \frac{\sigma}{c^2} \mathbf{H}(\mathbf{H}\mathbf{v}), \; \frac{\partial^2}{\partial n^2} \left(\varphi - \alpha T - \frac{c}{\sigma} p\gamma\right) - \frac{1}{c}\left(\mathbf{H} \times \frac{\partial \mathbf{v}}{\partial n}\right) = 0 \end{aligned} \tag{12}$$

(n is the distance reckoned along a normal to the wall).

The first equation of system (12) was obtained by substituting Ohm's law into the momentum equation, while the second is the continuity equation for the electric current. We assume that the velocity component normal to the surface in the boundary layer is equal to zero and that the tangential components of the gradients of potential, temperature, and pressure coincide with their values in the core of the flow.

We introduce the notation:

$$\mathbf{u} = \mathbf{v} - \mathbf{v}_0,\ \Phi = \varphi - \varphi_0,\ Q = T - T_0,\ P = p - p_0,$$

where v_0, φ_0, T_0, and p_0 are velocity, potential, temperature, and pressure in the flow core. Having integrated the second equation of (12) over the thickness of the boundary layer, we find

$$\frac{\partial}{\partial n}\left(\Phi - \alpha Q - \frac{c}{\sigma}\gamma P\right) = \frac{1}{c}(\mathbf{H} \times \mathbf{u})_n. \tag{13}$$

Taking Eqs. (12) and (13) into account, we obtain the following equation for u

$$\mu \frac{\partial^2 \mathbf{u}}{\partial n^2} = \mathrm{grad}(1 + \varkappa H^2)P + \frac{\sigma}{c^2} H_n^2 \mathbf{u} - \frac{\sigma}{c^2} \mathbf{H}(\mathbf{H}\mathbf{u})\mathbf{n}, \tag{14}$$

where **n** is a vector normal to the surface.

Since $u_n = 0$, then by projecting Eq. (14) on the normal to the surface we obtain the equality

$$(1 + \varkappa H^2)\frac{\partial P}{\partial n} = \frac{\sigma}{c^2} H_n(\mathbf{H}\mathbf{u}). \tag{15}$$

Projection of Eq. (14) on a plane tangent to the wall gives us the relation

$$\mu \frac{\partial^2 \mathbf{u}}{\partial n^2} = \frac{\sigma}{c^2} H_n^2 \mathbf{u}. \tag{16}$$

Equation (16) agrees completely with the equation for a Hartmann layer obtained in [1] without allowance for heat flow and the Hall effect. Thus, using these results, we can write the boundary conditions for the flow core:
on the nonconducting wall

$$\mathrm{sign} H_n \sqrt{\sigma\mu}\,(\mathrm{rot}\ \mathbf{v})_n + j_n = 0 \tag{17}$$

for the electrode

$$\varphi - \alpha T - \gamma \frac{c}{\sigma} p = \varphi^* - \alpha T^* - \gamma \frac{c}{\sigma} p^*. \tag{18}$$

Here, φ^*, T^*, and p^* are potential, temperature, and pressure on the surface of the electrode.

As a simple example, we will examine a flow of gas in a circular tube of constant cross section. The streamlines are rectilinear at high Hartmann numbers.

We assign a constant external magnetic field $H = H_0 e_z$ (H_0 = const) in the yz plane perpendicular to the direction of flow along the Ox axis. In this case, we write the general solution of (9-11) in the form

$$\begin{gathered} \sigma(\varphi - \alpha T) = A(y)z + B(y), \\ v_x = -\left(\frac{c^2}{\sigma H^2} + \varkappa \frac{c}{\sigma}\right)\frac{\partial p}{\partial x} - \frac{c}{H}\frac{\partial}{\partial y}\left(\varphi - \alpha T - \gamma \frac{c}{\sigma} p\right), \\ v_y = 0,\ v_z = 0,\ Q^0 = \frac{\partial p}{\partial x} = \mathrm{const}. \end{gathered} \tag{19}$$

We will assume that the process is isothermal, since $T = T^* = \text{const}$, while the boundaries of the tube, given by the equations $z = \pm \sqrt{R^2 - y^2}$, are electrodes with the potentials $\varphi_1^* = \text{const}$, $\varphi_2^* = \text{const}$. We find from the condition $v_y = 0$ that the pressure distribution in the given case satisfies the relation

$$p = Q^0 x + \frac{Q^0 H\gamma}{1 + H^2\gamma\varkappa} y + p^0, \; p^0 = \text{const}.$$

We use system (19) to find the distribution of potential

$$\sigma\varphi = \frac{\sigma(\varphi_1^* - \varphi_2^*)}{2\sqrt{R^2 - y^2}} + \frac{\sigma(\varphi_1^* + \varphi_2^*)}{2}.$$

Knowing the potential, temperature, and pressure, we determine the velocity of the flow in the tube v_x. We similarly solve the given problem in the case when the walls of the tube are nonconducting and the case when one wall is an electrode and the other wall an insulator.

Let us examine the flow of a conducting medium when $\mathbf{H} = H_0\mathbf{e}_z$, where $H_0 = \text{const}$ and $\mathbf{e}_z$ is a unit vector on the z axis of the cartesian coordinate system. Let the boundaries of the region be given by the equations $z = \pm f(x, y)$. Considering the symmetry of the problem relative to the plane $z = 0$ and examining processes for which $\rho = \rho(x, y)$, $\varphi = \varphi(x, y$, (p = p(x, y), $T = T(x, y)$, we can use Eqs. (7-8), (10-11) to obtain the system

$$\begin{gathered} j_x = -\frac{c}{H}p_y,\; j_y = \frac{c}{H}p_x,\; j_z = 0, \\ v_x = \left(-\frac{c^2}{\sigma H^2} - \frac{c^2}{\sigma}\varkappa\right)p_x + \frac{c}{H}\left(\varphi - \alpha T - \gamma\frac{c}{\sigma}p\right)_y,\; v_y = \left(-\frac{c^2}{\sigma H^2} - \frac{c^2}{\sigma}\varkappa\right) \times \\ p_y - \frac{c}{H}\left(\varphi - \alpha T - \gamma\frac{c}{\sigma}p\right)_x,\; v_z = \frac{c^2}{\sigma H^2}\Delta pz + \frac{c^2}{\sigma}\varkappa\Delta pz + \\ \frac{c}{\rho H}\left(\varphi_x - \alpha T_x - \gamma\frac{c}{\sigma}p_x\right)\rho_y - \left(\varphi_y - \alpha T_y - \gamma\frac{c}{\sigma}p_y\right)\rho_x + \\ + \frac{1}{\rho}\mathrm{grad}\rho\,\mathrm{grad}\left(\frac{c^2}{\sigma H^2} + \frac{c^2\varkappa}{\sigma}\right)z. \end{gathered} \tag{20}$$

We designate the partial derivatives as $p_x = \partial p/\partial x$, etc. In this case, we write the hydrodynamic boundary condition expressing the impermeability of the boundaries $v_n = 0$ in the form

$$(v_z - v_x f_x - v_y f_y)\big|_{z = f(x,y)} = 0.$$

We designate $\theta = \varphi - \alpha T - \gamma(c/\sigma)p$. Then having inserted the equations of system (20) into the boundary condition, we obtain the relation

$$\begin{gathered} \left(\frac{c^2}{\sigma H^2} + \frac{c^2}{\sigma}\varkappa\right)f\Delta p + \left(\left(\frac{c^2}{\sigma H^2} + \frac{c^2}{\sigma}\varkappa\right)p_x - \frac{c}{H}\theta_y\right)f_x + \left(\left(\frac{c^2}{\sigma H^2} + \frac{c^2}{\sigma}\varkappa\right)p_y + \frac{c}{H}\theta_x\right)f_y + \frac{cf}{\rho H}(\theta_x\rho_y - \theta_y\rho_x) + \\ \left(\frac{c^2}{\sigma H^2} + \frac{c^2}{\sigma}\varkappa\right)f\,\mathrm{grad}\,\rho\,\mathrm{grad}\,p = 0. \end{gathered} \tag{21}$$

If the boundaries of the region $z = \pm f(x, y)$ are insulators, then after inserting Eqs. (20) into Eq.(17) we obtain

$$\begin{gathered} \Delta\theta + \frac{\partial}{\partial y}\left(\frac{1}{\rho}(\theta_x\rho_y - \theta_y\rho_x)ff_x + \left(\frac{c}{\sigma H} + \frac{c}{\sigma}H\varkappa\right)\Delta pff_x + \right. \\ \left.\frac{1}{\rho}\,\mathrm{grad}\,\rho\,\mathrm{grad}\,p\left(\frac{c}{\sigma H} + \frac{cH\varkappa}{\sigma}\right)f\right) - \frac{\partial}{\partial x}\left(\frac{1}{\rho}(\theta_x\rho_y - \theta_y\rho_x)ff_y + \right. \\ \left.\left(\frac{c}{\sigma H} + \frac{cH\varkappa}{\sigma}\right)\Delta pff_y + \frac{1}{\rho}\,\mathrm{grad}\,\rho\,\mathrm{grad}\,p\left(\frac{c}{\sigma H} + \frac{cH\varkappa}{\sigma}\right)f\right) = \frac{(p_y f_x - p_x f_y)}{\sqrt{\sigma\mu}}. \end{gathered} \tag{22}$$

We will assume that the parameter $\varkappa \lesssim 1/H^2$. Then taking Eqs. (21) and (22) and discarding terms of the order 1/M relative to the other terms, we obtain

$$\frac{1}{\rho}(\theta_x\rho_y - \theta_y\rho_x) + \frac{1}{f}(\varphi_x f_y - \varphi_y f_x) = 0, \theta = \theta(\rho f),$$
$$\Delta\theta + \frac{\partial}{\partial y}\left(\frac{1}{\rho}(\theta_x\rho_y - \theta_y\rho_x)ff_x\right) - \frac{\partial}{\partial x}\left(\frac{1}{\rho}(\theta_x\rho_y - \theta_y\rho_x)ff_y\right) = \frac{(p_y f_x - p_x f_y)}{\sqrt{\sigma\mu}}. \tag{23}$$

The estimates made in the construction of system (23) correspond to removal of the terms containing pressure from the expressions for the components of velocity. In this case, the expressions for the velocity components taken the form

$$v_x = \frac{c}{H}\theta_y, v_y = -\frac{c}{H}\theta_x, v_z = \frac{c}{\rho H}(\theta_x\rho_y - \theta_y\rho_x). \tag{24}$$

It follows from the above results that for flows of gas in strong magnetic fields, the conditions under which the Hall effect can be ignored for any boundary conditions are

$$\varkappa \ll \frac{1}{H^2}, p \ll \frac{\sigma}{\gamma c}(\varphi - \alpha T).$$

Let us examine the flow of a conducting gas $p = p(\rho, T)$ in a region bounded by nonconducting walls $z = \pm f(r)$ ($r = \sqrt{x^2 + y^2}$). The region also contains electrodes with assigned constant potentials φ_1^* and φ_2^* ($\varphi_1^* \neq \varphi_2^*$), constant temperatures T_1^* and T_2^* ($T_1^* \neq T_2^*$), and constant pressures p_1^* and p_2^* ($p_1 = p_2^*$) on the surface of the electrodes. The electrodes are cylinders with generatrices parallel to the z axis and directrices $f_1 = \text{const}$, $f_2 = \text{const}$, $f_1 \neq f_2$.

We will assume that a constant temperature and pressure is maintained in the regions between the electrodes and insulators. Also, as was shown in [2], in this case we have the equality

$$\varphi'(f) = 0, \varphi(f) = \text{const},$$

from which we find that $\theta'(f) = 0$, $\theta(f) = \text{const}$, i.e., gas velocity is zero in this region. Thus, motion of the gas will take place only in the region between the two electrodes, i.e., the geometric structure of the flow will be planar and, accordingly, $v_z = cz/\rho H(\theta_x\rho_y - \theta_y\rho_x$ leave) in = 0:

$$\theta_x\rho_y - \theta_y\rho_x = 0, \theta = \theta(\rho). \tag{25}$$

Condition (25) is equivalent to the equality

$$\frac{d\rho}{dt} = \frac{c}{H}(\rho_x\theta_y - \rho_y\theta_x) = 0.$$

With allowance for Eqs. (23), we have

$$\theta = \theta(f), \rho = \rho(f).$$

Let us examine barotropic processes $p = p(\rho) = p(f)$.

System (23) then reduces to the form

$$\theta = \theta(f), f = f(r), \Delta\theta = 0,$$

from which we obtain

$$\theta = R_1 \ln \frac{1}{\sqrt{1 - f^2}} + R_2,$$

where the constants R_1 and R_2 are found from the boundary conditions

$$\theta(f_1) = \varphi_1^* - \alpha T_1^* - \gamma \frac{c}{\sigma} p_1^* = \text{const},$$
$$\theta(f_2) = \varphi_2^* - \alpha T_2^* - \gamma \frac{c}{\sigma} p_2^* = \text{const}.$$

Since $\rho = \rho(f)$, $T = T(f)$, $\varphi = \varphi(f)$, and $p = p(f)$, the streamlines $f = \text{const}$ are simultaneously isochores, isotherms, isobars, and lines of equal potential. Knowing the function $\theta = \theta(f)$, we can use Eq. (24) to find the velocity of the gas.

Generalizing the results we have obtained, we can state the following as regards thermomagnetohydrodynamic flows of gas in strong magnetic fields with allowance for the Hall effect in the case when the flow is bounded by nonconducting walls $z = \pm f(x, y)$.

1. For the parameters $\varkappa \lesssim 1/H^2$, plane flows of a conducting gas occur along the lines $f = \text{const}$ and are determined by the stream function

$$\theta = \theta(f) = \varphi - \alpha T - \gamma \frac{c}{\sigma} p.$$

2. The density of the gas is constant along the lines

$$f = \text{const, i.e. } \rho = \rho(f).$$

3. The distribution of density satisfies the equation

$$\frac{d\rho}{dt} = 0.$$

Let us consider the case of a variable potential magnetic field $\mathbf{H} = \text{grad } a$ and walls of arbitrary form. Following [2], we introduce the curvilinear coordinate system x, y, a, with basis vectors e_1, e_2, e_3 such that the coordinate lines x and y are orthogonal to the lines a. Lines a coincide with the magnetic lines of force.

The metric is determined by the components g_{ij} of the metric tensor

$$ds^2 = g_{11}dx^2 + 2g_{12}dxdy + g_{22}dy^2 + \frac{D}{H_0^2}da^2,$$
$$D = g_{11}g_{22} - g_{12}^2,\ H_0 = \text{const}. \tag{26}$$

Using the last equality of Eqs. (26) and taking into account the potential nature of the nature of the magnetic field $\mathbf{H} = \text{grad } a$, we see that $H\sqrt{D} = H_0$. We write the equations of the nonconducting walls in the form $a = H_0 f^+(x, y)$, $a = H_0 f^-(x, y)$. As in the above-examined case H = const, we ignore the terms containing pressure in the expressions for the velocity components. We will henceforth consider the case when $\varphi = \varphi(x, y)$, $\rho = \rho(x, y)$, $T = T(x, y)$. The expression for velocity takes the form

$$\mathbf{v} = \frac{c}{H_0}\theta_y e_1 - \frac{c}{H_0}\theta_x e_2 + \frac{cH}{H_0^2}\left(\theta_x F_y - \theta_y F_x + \frac{F}{\rho}(\theta_x \rho_y - \theta_y \rho_x)\right).$$

The condition of impermeability $v_n = 0$ on the walls gives

$$Df_x^{\pm}\theta_y - Df_y^{\pm}\theta_x - (\theta_x F_y^{\pm} - \theta_y F_x^{\pm} + \frac{F}{\rho}(\theta_x \rho_y - \theta_y \rho_x) + \frac{1}{\rho}C(x,y)) = 0,$$
$$F^{\pm} = F^{\pm}(x,y,H_0 f^{\pm}),\ F = \frac{1}{H_0}\int D(x,y,a)da. \tag{27}$$

Subtracting Eq. (27) with minus superscripts from the same equation with plus superscripts, we obtain

$$2(\theta_x\psi_y - \theta_y\psi_x) + \frac{\psi}{\rho}(\theta_x\rho_y - \theta_y\rho_x) = 0,$$
$$\psi = F^+ - F^- = \frac{1}{H_0}\int D da. \tag{28}$$

We find from Eq. (28) that

$$\theta = \theta(\rho\psi^2).$$

It the gas flow takes place in the direction perpendicular to the magnetic field, the $\theta = \theta(\psi)$ and $\rho = \rho(\psi)$ and motion will occur along the surface $\psi(x, y) = \text{const}$ – which in the given case are isochores. The pressure distribution can be bound by the method described in [2].

The results obtained here are also valid for liquids, but in this case Ohm's law will have the form

$$\mathbf{j} = \sigma\left[\left(\operatorname{grad}\varphi + \frac{\mathbf{v}}{c}\times\mathbf{H}\right) - \alpha\operatorname{grad}T\right] - \gamma(\mathbf{j}\times\mathbf{H}).$$

As an example, let us examine the case of plane flows of a nonuniform liquid and walls of arbitrary conductivity.

For plane flows of a nonuniform liquid with $d\rho/dt = 0$ in a constant magnetic field $\mathbf{H} = H_0\mathbf{e}_z$ perpendicular to the plane of the flow (x, y), where $\varphi = \varphi(x, y)$, $\rho = \rho(x, y)$, $T = T(x, y)$, $p = p(x, y)$, we obtain the following from Eq. (10) at $\varkappa = 0$

$$v_z = \frac{c^2 z}{\sigma H^2}\Delta p = 0.$$

We then find from this equation that the pressure distribution satisfies the Laplace equation

$$p_{xx} + p_{yy} = \Delta p = 0, \tag{29}$$

i.e., we will assume that pressure p can be found from Eq. (29) and we will take p to be a known function.

We now introduced the stream function β for the pressure gradient so that

$$p_x = -\frac{\sigma H}{c}\beta_y,\ p_y = \frac{\sigma H}{c}\beta_x.$$

Thus, the components of velocity take the form

$$v_x = \frac{c}{H}\left(\varphi - \alpha T - \gamma\frac{c}{\sigma}p + \beta\right)_y,$$
$$v_y = -\frac{c}{H}(\varphi - \alpha T - \gamma\frac{c}{\sigma}p + \beta)_x. \tag{30}$$

We designate $\varepsilon = \varphi - \alpha T - \gamma(c/\sigma)p + \beta$. Using the condition of impermeability of the boundaries $v_n = v_x f_x + v_y f_y$, we find that $\varepsilon = \varepsilon(f)$, i.e., the velocity vector always coincides in terms of direction with the tangent to the curve $f(x, y) = \text{const}$. We find from the equation $d'/dt = 0$ that density is constant along the lines $f = \text{const}$, i.e., $\rho = \rho(f)$. This means that arbitrary plane flows of a nonuniform conducting liquid in strong magnetic fields have the following properties:

1) the pressure distribution satisfies the Laplace equation $\Delta p = 0$;

2) flow of the liquid always occurs along the lines $f = \text{const}$ and is determined by the stream function

$$\varepsilon = \varphi - \alpha T - \gamma\frac{c}{\sigma}p + \beta;$$

3) the density of the liquid is constant along the lines f = const, i.e., $\rho = \rho(f)$.

Knowing the pressure distribution, we can find the current density. From the condition $v_n = 0$ we have

$$(\varphi - \alpha T - \gamma \frac{c}{\sigma}p)_x f_y - (\varphi - \alpha T - \gamma \frac{c}{\sigma}p)_y f_x =$$

$$\theta_x f_y - \theta_y f_x = -\frac{c}{\sigma H}(p_x f_x + p_y f_y).$$

Using the equality $d\theta/dl = (\theta_y f_x - \theta_x f_y) 1/\sqrt{f_x^2 + f_y^2}$, where l is a length reckoned along the curve f = const, we obtain

$$\theta = \frac{c}{\sigma H}\int (p_x f_x + p_y f_y)\frac{1}{\sqrt{f_x^2 + f_y^2}}dl.$$

Thus, the velocity of the liquid is determined from system (30).

In the examples examined earlier, the geometric structure of the flow was explicitly dependent on the form of the bounding surface $z = \pm f(x, y)$, i.e., we obtained a flow with a stable, organized structure that could be controlled. Such flows have the form of a steady vortex with streamlines f = const. Thus, we have a new direction in continuum mechanics – the controlled organization of flow structures with external fields and specially chosen bounding surfaces.

Instead of the static confinement of a partially ionized gas in strong magnetic fields with allowance for the Hall effect and heat flow, we can examine the problem of organizing slow, controlled steady motions of a plasma within a closed volume, i.e., we can impart a certain rotation mechanism to a gas along specified lines f = const and prevent it from dispersing.

As regards flows of liquid metals, the results obtained here may find application in fusion-reactor cooling systems, as well as in the production of tritium during neutron bottombardment in various controlled fusion reactors in which heat flow and the Hall effect are significant (an example would be the blanket of the Mark-IIA reactor at the Kalemsk laboratory).

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